Technical Comments

Comment on "A Comparison of Solutions to a Blunt Body Problem"

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Dr. Van Tuyl of the Naval Ordnance Laboratory (NOL) informs me that the expression "correction to its formulation," paragraph 6[1] is misleading. Only an algebraic error was corrected in his equations to give the improved NOL curve in Fig. 1.

Reference

[1] Perry, J. C. and Pasiuk, L., "A comparison of solutions to a blunt body problem," AIAA J. 4, 1425–1426 (1966).

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Comments on a Proposed Variation of the Cohen-Reshotko Method in Boundary-Layer Theory

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A variation in the method developed by Cohen and Reshotko[1] for calculating laminar boundary-layer properties in a compressible flow with streamwise pressure gradients was recently presented by N. Ness.[2] To the present authors, his results appear to be rather unsatisfactory in that they seem to fail to account for all of the significant parameters. It is the purpose of the present comments to discuss this shortcoming, and to recommend changes that yield a revised method that should be more universally acceptable.

The Cohen-Reshotko method requires knowledge of the distribution along the body of the so-called correlation number, $n(x)$. This is a parameter which is proportional to the square of the local momentum thickness, and this fact was used by Ness in outlining a new method for its determination. He used the Karman momentum integral equation, together with the concept of local similarity, to develop the following expression [Eq. (14) of Ref. 2] for the local momentum thickness (squared)

$$\theta(x)^2 = 2(1+\epsilon)\,\theta^2_{\text{stag}}\left(\frac{du_e}{dx}\right)_{\text{stag}} \times \left(1+\frac{\gamma-1}{2}M_e^2\right)^{(2-\gamma)/(\gamma-1)} \frac{\int_0^x p_e^{\alpha}u_e r_w^{2\epsilon}dx}{p_e^{\alpha}u_e^2 r_w^{2\epsilon}} \tag{1}$$

Since the preceding result was partially based on the assumption of local similarity, it is natural to compare it to what is obtained from local similarity exclusively. This result is given below:

$$\theta(x)^2 = 2(1+\epsilon)\,\theta^2_{\text{stag}}\left(\frac{du_e}{dx}\right)_{\text{stag}} \times \left(1+\frac{\gamma-1}{2}M_e^2\right)^{(2-\gamma)/(\gamma-1)} \frac{\theta_{\text{tr}}(x)^2}{\theta_{\text{tr}}(0)^2}\frac{\int_0^x p_e^{\alpha}u_e r_w^{2\epsilon}dx}{p_e^{\alpha}u_e^2 r_w^{2\epsilon}} \tag{2}$$

In the preceding expression, the quantity $\theta_{\text{tr}}(x)$ is the transformed momentum thickness defined as

$$\theta_{\text{tr}}(x) = \int_0^\infty f_\eta(1-f_\eta)d\eta \tag{3}$$

In the local similarity method, θ_{tr} depends on the local values of three variables: 1) the pressure gradient parameter β, defined by Eq. (4) of Ref. 2; 2) the wall-to-freestream stagnation temperature ratio, $T_w/T_{e_{\text{stag}}}$, and 3) a dissipation parameter, which is proportional to the square of the local Mach number M_e, if the Prandtl number is not one. The quantity θ_{tr} is evaluated from similar solutions to the momentum and energy equations, such as those presented in Refs. 3 and 4. Clearly, Eq. (2) differs from Ness' results by the appearance of the ratio $[\theta_{\text{tr}}(x)/\theta_{\text{tr}}(0)]^2$. This ratio can be interpreted as a correction factor which partially accounts for the effects of the variation of β, $T_w/T_{e_{\text{stag}}}$, and M_e along the body.

The fact that this correction factor is not present in Eq. (1) is what appears to be unsatisfactory, as can be illustrated by the following example. Consider a blunted wedge in supersonic flow where the ratio of nose radius to base height is very much less than one. It is expected that far back from the nose, the boundary-layer development is essentially similar to that on a flat plate. Calculations of the momentum thickness by Eq. (2) would bear this out, and in fact, in the limit of zero nose radius, this equation yields the flat plate result exactly. On the other hand, the momentum thickness calculated by Eq. (1) would differ by the factor $\theta_{\text{tr}}(x)/\theta_{\text{tr}}(0)$, which in this example, has a limiting value of about 1.6 when the wall is adiabatic, and 0.9 when the wall is very cold. These factors are squared in the correlation number, which is used to determine other physical characteristics such as heat transfer, and are thus quite significant. This example gives a rather clear indication that the parameter θ_{tr} is important. The next question of concern is why it does not appear in Eq. (1).

To answer this, the derivation outlined by Ness has been reviewed in detail and the following perhaps subtle point is noted. Ness states (correctly) that in the approximation of local similarity, the group of terms he calls F in the Karman integral equation becomes

$$F_{\text{loc sim}} = C\frac{T_e^2}{T_{\text{ref}}^2}\frac{p_{\text{ref}}^{\alpha}}{p_e^{\alpha}}\theta_{\text{tr}}^2 \tag{4}$$

He then "seeks an expression for F which, when approximated by local similarity, yields the aforementioned $F_{\text{loc sim}}$." This he chooses to write as

$$F = \frac{1}{2}\frac{1}{\xi}\frac{d\xi}{dx}\frac{u_e\theta^2}{\nu_{\text{ref}}} \tag{5}$$

where

$$\xi = \frac{C\rho_{\text{ref}}\mu_{\text{ref}}}{p_{\text{ref}}}\int_0^x p_e^{\alpha}u_e r_w^{2\epsilon}dx$$

and substitutes it into the Karman integral equation which,

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as a result, can be integrated to yield (with some additional manipulation) Eq. (1). An alternative expression for Eq. (4) can be written as

$$F_{\text{loc sim}} = \frac{1}{2}\,\frac{1}{\xi}\,\frac{d\xi}{dx}\left(\frac{u_e\theta^2}{\nu_{\text{ref}}}\right)_{\text{loc sim}} \tag{6}$$

and hence, by Eq. (5), F becomes Eq. (4) in the case of local similarity. However, it is not clear that Eq. (5) is a correct (and hence unique) choice for the form of F in general. If it is not, then quite a different answer could be obtained. Since Eq. (1) does give an erroneous result in the example discussed, it appears that the suitability of the choice is indeed questionable.

Rather than assuming the form for F, which is like assuming certain characteristics of the answer, suppose that F is simply replaced in the Karman integral equation by $F_{\text{loc sim}}$, i.e., by Eq. (4) or (6). In this way, the resulting solution may give what could be deemed a correction to local similarity. With this approximation, the Karman integral equation can be written as follows:

$$\frac{d}{dx}\left(\frac{u_e\theta^2}{\nu_{\text{ref}}}\right) = \frac{1}{\xi}\,\frac{d\xi}{dx}\left(\frac{u_e\theta^2}{\nu_{\text{ref}}}\right)_{\text{loc sim}} - \left\{\frac{d}{dx}\ln\left[u_e r_w^{2\epsilon} a_e^{4/(\gamma-1)}\right]\right\}\frac{u_e\theta^2}{\nu_{\text{ref}}} \tag{7}$$

This equation differs from that solved by Ness in that the quantity $u_e\theta^2/\nu_{\text{ref}}$ appearing in the first term on the right-hand side is evaluated from local similarity, rather than being the dependent variable of the differential equation. The solution to Eq. (7) can be expressed as follows:

$$\theta(x)^2 = 2(1+\epsilon)\theta_{\text{stag}}^2\left(\frac{du_e}{dx}\right)_{\text{stag}} \times \left(1+\frac{\gamma-1}{2}M_e^2\right)^{(2-\gamma)/(\gamma-1)} \cdot \left\{\frac{\theta_{\text{tr}}(x)^2}{\theta_{\text{tr}}(0)^2}\,\frac{\int_0^x p_e^{\alpha}u_e r_w^{2\epsilon}dx}{p_e^{\alpha}u_e^2 r_w^{2\epsilon}} - \frac{\int_0^x [d(\theta_{\text{tr}}^2)/dx]\left(\int_0^{x'} p_e^{\alpha}u_e r_w^{2\epsilon}dx''\right)dx'}{p_e u_e^2 r_w^{2\epsilon}\theta_{\text{tr}}(0)^2}\right\} \tag{8}$$

The first term corresponds to local similarity by comparison with Eq. (2), whereas the second term, which contains a derivative of θ_{tr} along the body, represents a correction to local similarity. The preceding result gives the correct limit for the example previously discussed, and since it contains θ_{tr}, it appears to be a more satisfying result than that given by Eq. (1).

Ness states that Eq. (1) has been used to calculate the boundary-layer development on a two-dimensional circular cylinder in incompressible flow, and that the results agreed favorably with the Blasius series solution. This same calculation has also been performed using Eq. (8), and the results were also favorable. However, it is not our primary concern that either of these calculations agree numerically within some acceptable percentage of more exact solutions for selected examples. (Certainly, such agreement does not constitute proof of the universal validity of either method.) What is essential to us is that any method that claims to be general must include all effects which are potentially important, and which can be shown to be important in limiting cases. The method of Ref. 2 has apparently failed in this respect. Hopefully, the difficulty has been overcome by the changes suggested here.

References

[1] Cohen, C. B. and Reshotko, E., "The compressible laminar boundary layer with heat transfer and arbitrary pressure gradient," NACA Lewis Flight Propulsion Lab. Rept. 1294 (1956).

[2] Ness, N., "Some comments on the laminar compressible boundary layer analysis with arbitrary pressure gradient," AIAA J. **4,** 330–331 (1966).

[3] Cohen, C. B. and Reshotko, E., "Similar solutions for the compressible laminar boundary layer with heat transfer and pressure gradient," NACA Lewis Flight Propulsion Lab. Rept. 1293 (1956).

[4] Cohen, N. B., "Boundary layer similar solutions and correlation equations for laminar heat transfer distribution in equilibrium air at velocities up to 41,100 feet per second," NASA Langley Research Center Rept. R-118 (1961).